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STATIC AND VIBRATION ANALYSIS OF ANISOTROPIC COMPOSITE LAMINATES BY FINITE STRIP METHOD

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Abstract—The finite strip method has been employed for the analysis of shear-deformable composite laminates since 1976. A number of works have shown the high accuracy, efficiency and convenience of this method.

In the present study, a new shear-deformable finite strip is developed to analyse the static and vibration behaviour of composite laminates. By selecting the proper displacement functions, the new strip includes all the effects of material anisotropy and suits arbitrary inplane boundary conditions at the ends. Thus, the present method is applicable not only to cross-ply laminates and symmetrical angle-ply laminates, but also to arbitrary angle-ply laminates, e.g. antisymmetrical angle-ply laminates and (0/45/−45/90 degree) laminates.

INTRODUCTION

Fibre-reinforced composite materials have been extensively used as structural members in a variety of industrial products and structures, such as rockets, aircraft, marine vessels, auto bodies, pipes, pressure vessels etc. by virtue of the high strength-to-weight ratio, excellent resistance to corrosive substances, satisfactory durability under fatigue loading and so forth. Creating more efficient analysis methods for composite structures has become an important target of many researchers.

For structures with regular geometry, e.g. a rectangular or sectorial plate with two opposite simply-supported sides, the finite strip method has proven to be a very efficient numerical method, Cheung (1976). This method uses a series of beam eigenfunctions to express the longitudinal variations of displacements. Thus, the two-dimensional analysis is transformed into a one-dimensional one. Consequently, the computer time and storage requirement are reduced significantly, and the input data preparation and output interpretation are simplified substantially. In addition, the numerical errors attributed to material anisotropy are cut down by considerable decrease in the number of unknowns involved in the analysis.

The finite strip method has been successfully employed for the analysis of shear-deformable composite laminates since 1976 (Hinton, 1976, 1977; Craig and Dawe, 1983; Azizian and Dawe, 1985; Dawe and Peshkam, 1989, etc.). All the references have shown the high accuracy, efficiency and convenience of this method.

In the present study, a new shear-deformable finite strip is developed to analyse the static and vibration behaviour of composite laminates. By selecting the proper displacement functions, the new strip includes all the effects of material anisotropy and suits arbitrary inplane boundary conditions at the ends. Thus, the present method is applicable not only to cross-ply laminates and symmetrical angle-ply laminates, but also to arbitrary angle-ply laminates, e.g. antisymmetrical angle-ply laminates and (0/45/−45/90 degree) laminates.

In this analysis, the transverse shear deformation is taken into consideration following the first order shear deformation theory based on the Mindlin assumptions. According to this theory, any straight line originally normal to the plate middle surface is assumed to remain straight, but not generally normal to the middle surface after deformation. In order

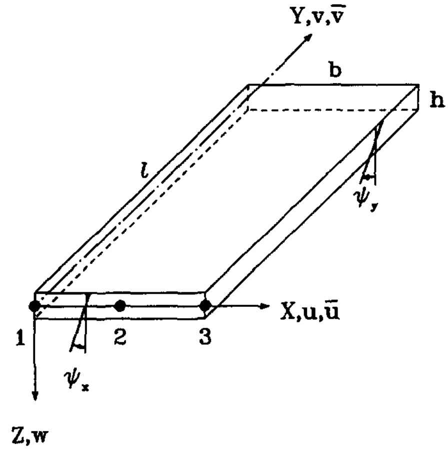

Fig. 1. A finite strip.

to simulate the flexural behaviour of both moderately thick plates and thin plates, reduced Gaussian integration is used in forming the stiffness matrix of the strip.

FINITE STRIP FORMULATION

In the present analysis, the rectangular composite laminate is simulated by a number of finite strips, each of which has 2–6 equally spaced nodal lines (Fig. 1). For the mth harmonic, the displacement parameters of nodal line i are

$$\{\delta\}_{im} = [u_{im}, \bar{u}_{im}, v_{im}, \bar{v}_{im}, w_{im}, \psi_{xim}, \psi_{yim}]^{\mathrm{T}}. \tag{1}$$

For the laminates with two simply-supported opposite sides, the boundary conditions at both ends of the strip are

$$w = \psi_x = 0, \quad M_y = 0, \quad N_y = N_{xy} = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad y = l. \tag{2}$$

In this case, the displacement field within a strip can be expressed as

$$\begin{aligned} u &= \sum_{m=1}^{r} \sum_{i=1}^{nd} N_i(x)(u_{im} - z\psi_{xim}) \sin\frac{m\pi y}{l} + \sum_{m=1}^{r} \sum_{i=1}^{nd} N_i(x)\bar{u}_{im} \cos\frac{m\pi y}{l}, \\ v &= \sum_{m=1}^{r} \sum_{i=1}^{nd} N_i(x)(v_{im} - z\psi_{yim}) \cos\frac{m\pi y}{l} + \sum_{m=1}^{r} \sum_{i=1}^{nd} N_i(x)\bar{v}_{im} \sin\frac{m\pi y}{l}, \\ w &= \sum_{m=1}^{r} \sum_{i=1}^{nd} N_i(x) w_{im} \sin\frac{m\pi y}{l} \end{aligned} \tag{3}$$

or in a compact form

$$\{f\} = [u, v, w]^{\mathrm{T}} = \sum_{m=1}^{r} \sum_{i=1}^{nd} [N]_{im}\{\delta\}_{im}, \tag{4}$$

where r is the number of harmonics employed in the analysis, nd is the number of nodal lines in each finite strip, l is the length of the strip, $N_i(x)$ is the transverse shape function for nodal line i and has the following form:

$$N_i(x) = \prod_{\substack{j=1 \\ j\neq i}}^{nd} \frac{x-x_j}{x_i-x_j} \tag{5}$$

and $[N]_{im}$ is the displacement matrix written as

$$[N]_{im} = \begin{bmatrix} N_i(x)S & N_i(x)C & 0 & 0 & 0 & -zN_i(x)S & 0 \\ 0 & 0 & N_i(x)C & N_i(x)S & 0 & 0 & -zN_i(x)C \\ 0 & 0 & 0 & 0 & N_i(x)S & 0 & 0 \end{bmatrix}, \tag{6}$$

in which $S = \sin k_m y$, $C = \cos k_m y$ and $k_m = m\pi/l$.

The following strain–displacement relationships are used in the analysis:

$$\varepsilon_x = \frac{\partial u}{\partial x},$$
$$\varepsilon_y = \frac{\partial v}{\partial y},$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y},$$
$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}. \tag{7}$$

By substituting (3) into (7), the strain vectors can be expressed in terms of displacement parameters as

$$\{\varepsilon\} = [\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}]^{\mathrm{T}} = \sum_{m=1}^{r} \sum_{i=1}^{nd} [B]_{im}\{\delta\}_{im}, \tag{8}$$

where $[B]_{im}$ is the strain matrix with the following expression:

$$[B]_{im} = \begin{bmatrix} N_{i,x}S & N_{i,x}C & 0 & 0 & 0 & -zN_{i,x}S & 0 \\ 0 & 0 & -k_mN_iS & k_mN_iC & 0 & 0 & zk_mN_iS \\ k_mN_iC & -k_mN_iS & N_{i,x}C & N_{i,x}S & 0 & -zk_mN_iC & -zN_{i,x}C \\ 0 & 0 & 0 & 0 & k_mN_iC & 0 & -N_iC \\ 0 & 0 & 0 & 0 & N_{i,x}S & -N_iS & 0 \end{bmatrix} \tag{9}$$

in which $N_{i,x} = \mathrm{d}N_i(x)/\mathrm{d}x$.

It is assumed that the laminate is manufactured from orthotropic layers (or plies) of preimpregnated unidirectional fibrous composite materials. Neglecting σ_z, for each layer, the stress–strain relationship in the x–y–z coordinate system can be stated as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{26} & 0 & 0 \\ Q_{16} & Q_{26} & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} \\ 0 & 0 & 0 & Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{10}$$

or

$$\{\sigma\} = [Q]\{\varepsilon\},$$

where Q_{ij} for $i, j = 1, 2, 6$ are plane-stress reduced stiffnesses and Q_{ij} for $i, j = 4, 5$ are transverse shear stiffnesses.

Following the procedure commonly used in the finite strip, analysis yields the stiffness matrix and the mass matrix of the strip. The submatrices corresponding to nodal lines i and j can be evaluated as follows:

$$[K]_{ijmn} = \int_h \int_l \int_b [B]^{\mathrm{T}}_{im}[Q][B]_{jn}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z, \tag{11}$$

$$[M]_{ijmn} = \int_h \int_l \int_b \rho[N]^{\mathrm{T}}_{im}[N]_{jn}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z, \tag{12}$$

where b and h are the width and the thickness of the strip respectively, ρ is the density of the composite material, and m and n denote the related series terms.

The integrations in (11) and (12) can be carried out analytically in the y and z directions, and the following expressions may be useful:

$$I_1 = \int_l \sin k_m y \sin k_n y\,\mathrm{d}y = \begin{cases} \dfrac{l}{2} & \text{for} \quad m = n \\ 0 & \text{for} \quad m \neq n \end{cases}$$

$$I_2 = \int_l \cos k_m y \cos k_n y\,\mathrm{d}y = \begin{cases} \dfrac{l}{2} & \text{for} \quad m = n \\ 0 & \text{for} \quad m \neq n \end{cases}$$

$$I_3 = \int_l \sin k_m y \cos k_n y\,\mathrm{d}y = \begin{cases} \dfrac{2ml}{\pi(m^2-n^2)} & \text{for} \quad m-n = 2k+1 \\ 0 & \text{for} \quad m-n = 2k \\ & \quad (k = 0, 1, 2, \ldots) \end{cases} \tag{13}$$

and

$$A_{ij} = \int_h Q_{ij}\,\mathrm{d}z = \sum_{k=1}^{n_k} Q_{ij}[z_k - z_{k-1}], \quad i, j = 1, 2, 6,$$

$$A_{ij} = \int_h Q_{ij}\,\mathrm{d}z = k_i k_j \sum_{k=1}^{n_k} Q_{ij}[z_k - z_{k-1}], \quad i, j = 4, 5,$$

$$B_{ij} = \int_h z Q_{ij}\,\mathrm{d}z = \frac{1}{2}\sum_{k=1}^{n_k} Q_{ij}[z_k^2 - z_{k-1}^2], \quad i, j = 1, 2, 6,$$

$$D_{ij} = \int_h z^2 Q_{ij}\,\mathrm{d}z = \frac{1}{3}\sum_{k=1}^{n_k} Q_{ij}[z_k^3 - z_{k-1}^3], \quad i, j = 1, 2, 6, \tag{14}$$

where n_k is the number of layers in the laminate, z_{k-1} and z_k are the vertical coordinates of the two faces of the kth layer, k_4 and k_5 are the shear correction factors.

In order to eliminate the "shear locking" effect in the analysis of thin plates, in the x direction, the integrations in (11) and (12) are implemented using the reduced integration technique corresponding to the use of $(nd-1)$ Gauss points across the strip.

The load vector for the mth harmonic and the ith nodal line equivalent to a distributed vertical load p_z is

$$\{P\}_{im} = [0,0,0,0,Z_{im},0,0]^{\mathrm{T}} \tag{15}$$

in which

$$Z_{im} = \int_l \int_b p_z N_i(x) \sin k_m y \, \mathrm{d}x \, \mathrm{d}y. \tag{16}$$

Because the integral I_3 does not always vanish for $m \neq n$, different series terms are coupled in some cases, as described by Dawe and Peshkam (1989).

After assembling the above strip matrices over the entire structure, the deflections of the laminate under external loading, $\{\delta\}$, its natural frequencies, ω, and the mode shapes of the free vibration, $\{\bar{\delta}\}$, can be obtained by solving the following matrix equations using standard computer sub-programs:

$$[K]\{\delta\} = \{P\}, \tag{17}$$

$$[K]\{\bar{\delta}\} = \omega^2[M]\{\bar{\delta}\}. \tag{18}$$

NUMERICAL EXAMPLES

Six examples are presented to highlight the present approach. In the analytical solutions, which are compared with the present results in the following examples, $k_4^2 = k_5^2 = 5/6$ was used. Therefore, the same value is also taken for the present analysis. And in all the six examples only the quadratic strips (with three nodal lines each) are employed. This type of strip is very efficient and successfully applied to the flexural analysis of orthotropic composite laminates by Hinton since 1975.

1. *Four layer (0/90/90/0) square cross-ply laminated plate under sinusoidal loads*

A square laminate of side length a, and thickness h, is composed of four equally thick layers oriented at (0/90/90/0 degrees). It is simply supported on all the edges and subjected to a sinusoidal vertical pressure of the following form:

$$p_z = P \sin\frac{\pi x}{a} \sin\frac{\pi y}{a}$$

with the origin of the coordinate system being located at the lower left corner on the midplane.

The lamina properties are assumed to be:

$$E_1 = 25.0E_2, \quad G_{12} = G_{13} = 0.5E_2, \quad G_{23} = 0.2E_2 \quad \text{and} \quad \nu_{12} = 0.25,$$

where 1 is the fiber direction, 2 is the transverse to fiber inplane direction, respectively, and 3 refers to the direction normal to plate midplane.

By virtue of the symmetry of the deformation, only half the laminate is to be analysed. One, two or three quadratic strips and one harmonic are employed in each analysis.

Table 1. (0/90/90/0) square laminated plate under sinusoidal load

a/h		$(wh^3E_2/Pa^4)\times 10^2$ at $[a/2, a/2, 0]$	$\sigma_x(h^2/Pa^2)$ at $[a/2, a/2, h/2]$	$\sigma_y(h^2/Pa^2)$ at $[a/2, a/2, h/4]$	$\tau_{xy}(h^2/Pa^2)$ at $[0, 0, h/2]$
10	1 strip	0.6629	0.5689	0.3617	0.0261
	2 strips	0.6627	0.5220	0.3615	0.0248
	3 strips	0.6627	0.5097	0.3615	0.0244
	FSDT	0.6628	0.4989	0.3615	0.0241
20	1 strip	0.4890	0.6014	0.2947	0.0237
	2 strips	0.4910	0.5518	0.2957	0.0226
	3 strips	0.4911	0.5388	0.2957	0.0224
	FSDT	0.4912	0.5273	0.2957	0.0221
100	1 strip	0.4307	0.6138	0.2690	0.0227
	2 strips	0.4335	0.5632	0.2705	0.0218
	3 strips	0.4336	0.5499	0.2705	0.0215
	FSDT	0.4337	0.5382	0.2705	0.0213
	CPT	0.4312	0.539	0.269	0.0213

The resulting dimensionless deflections of the center and the maximum bending stresses are listed in Table 1 in comparison with those from the first order shear deformation theory (FSDT) (Reddy and Chao, 1981), and from the classical plate theory (CPT).

It can be seen that for the length-to-thickness ratios, a/h, from 10 to 100, the present solutions converge to the Mindlin plate theory very fast. Only one strip gives accurate results of maximum deflections with error less than 0.5%. The accuracy of stresses is also satisfactory, only two strips yield the maximum bending stresses of errors within 4.6%.

2. *Two layer* $(\theta/-\theta)$ *square angle-ply laminated plate under sinusoidal load*

A square laminate of side length a, and thickness h, consists of two equally thick layers oriented at $(\theta/-\theta)$. Its four edges are hinged and free in the tangential direction but immovable in the normal direction.

The lamina properties are assumed to be:

$$E_1 = 40.0E_2, \quad G_{12} = G_{13} = 0.6E_2, \quad G_{23} = 0.5E_2 \quad \text{and} \quad \nu_{12} = 0.25.$$

The load is the same sinusoidal distributed one as used in the previous example.

In the analysis, the entire plate is divided into two quadratic strips, and only one longitudinal harmonic is required.

The resulting dimensionless deflections of the center for different values of a/h and θ are given in Table 2 and compared with the solutions of the thick plate theory (FSDT) (Whitney and Pagano, 1970), and with the classical plate theory (CPT).

Table 2. $(\theta/-\theta)$ square laminated plate under sinusoidal load

		$10^2wh^3E_2/Pa^4$			
a/h		$\theta = 0°$	$\theta = 15°$	$\theta = 30°$	$\theta = 45°$
10	Present	0.4564	0.6041	0.6108	0.5803
	FSDT	0.4581	0.6053	0.6099	0.5773
20	Present	0.3227	0.4926	0.5222	0.4962
	FSDT	0.3253	0.4949	0.5224	0.4944
50	Present	0.2845	0.4609	0.4973	0.4726
	FSDT	0.2875	0.4636	0.4979	0.4711
100	Prsent	0.2790	0.4564	0.4938	0.4692
	FSDT	0.2820	0.4591	0.4944	0.4678
	CPT	0.280	0.458	0.493	0.467

Table 3. Fundamental frequency of angle-ply laminated plate of $a/h = 10$

No. of layers	$\omega a^2(\rho/E_2h^2)^{0.5}$			
	$\theta = 30°$		$\theta = 45°$	
	Present	Bert and Chen	Present	Bert and Chen
2	12.79	12.68	13.13	13.04
4	17.77	17.63	18.58	18.46
8	18.56	18.42	19.40	19.29
16	18.74	18.60	19.59	19.48

It can be noted that for all the listed a/h and θ, the present analysis is in close agreement with the analytical solutions.

3. *Free vibration of square anti-symmetrical angle-ply laminated plate*

A square plate of side length a, thickness h, and density ρ, is laminated of an even number of equally thick layers which are alternatively oriented at angles θ and $-\theta$. The length-to-thickness ratio is $a/h = 10$.

The boundary conditions and the lamina properties are identical to those of Example 2.

The effects of the lamination angle and the number of layers on the dimensionless fundamental frequency are shown in Table 3. The results obtained by using two quadratic strips and one harmonic give a good comparison with the thick plate theory solutions (Bert and Chen, 1978).

4. *Effect of the bending-twisting coupling*

A square laminate of $a/h = 20$ is constructed from four equally thick layers oriented at $(45/-45/-45/45$ degrees). Its four edges are hinged and free to move in the normal inplane direction but immovable in the tangential direction. It can be readily found that $B_{ij} = 0$ $(i, j = 1, 2, 6)$ but D_{16} and D_{26} do not vanish, i.e. there exists bending-twisting coupling but no bending-inplane coupling.

The material properties of each layer are as follows:

$$E_1 = 14.0E_2, \quad G_{12} = G_{13} = 0.533E_2, \quad G_{23} = 0.323E_2 \quad \text{and} \quad \nu_{12} = 0.30.$$

The laminate is simulated by 2 and 4 quadratic strips respectively. The resulting deflections of the centre and the bending moments due to uniform distributed load q_0 are listed in Table 4 in comparison with the analytical solution (Whitney, 1987) and with the orthotropic solution which is obtained by neglecting bending-twisting coupling.

Table 4. $(45/-45/-45/45$ degree) square laminated plate under uniform load

No. of strips	No. of terms	$10^3wh^3E_2/q_0a^4$	$10^2M_x/q_0a^2$	$10^2M_y/q_0a^2$
2	2	8.153	4.522	4.685
4	2	7.982	3.881	3.935
	3	8.107	4.079	4.149
	4	8.150	4.174	4.213
Analytical solution		8.090	4.175	
Orthotropic solution		6.915	3.674	

Table 5. Fundamental frequency of rectangular (45/−45/−45/45) plate

	$\omega a^2(\rho/E_2h^2)^{0.5}$				
a/h	$b/a = 1$	$b/a = 2$	$b/a = 3$	$b/a = 4$	$b/a = 5$
10	12.716	7.849	6.704	6.276	6.073
20	14.074	8.346	6.928	6.568	6.342
50	14.551	8.505	7.155	6.658	6.425
100	14.623	8.529	7.171	6.671	6.437

Table 6. Convergence for long plates in Example 5, $b/a = 5$, $a/h = 20$

No. of	$\omega a^2(\rho/E_2h^2)^{0.5}$				
terms	1 strip	2 strips	3 strips	4 strips	8 strips
1	7.722	6.552	6.482	6.470	6.464
2	7.631	6.428	6.355	6.342	6.336
3	7.624	6.411	6.337	6.324	6.318
4	7.615	6.405	6.331	6.318	6.312
10	7.604	6.393	6.319	6.306	6.300

Table 7. (0/45/−45/90 degree) square laminated plate

a/h	$10^2\omega h^3E_2/1_0a^4$	$\omega a^2(\rho/E_2h^2)^{0.5}$
10	1.6406	9.67
20	1.4879	10.31
50	1.4356	10.55
100	1.4272	10.59

5. *Effect of aspect ratio* b/a *on free vibration of rectangular (45/−45/−45/45 degree) plate*

The free vibration of a rectangular (45/−45/−45/45 degree) laminate of side lengths a and b is analysed by the present method. The material properties and boundary conditions are identical to example 4.

A variety of strip divisions combined with different numbers of series terms is employed for the analysis. It is found that only four quadratic strips (parallel to longer sides) and two series terms are sufficient to give converged solutions for the fundamental frequency of the laminate and further increasing the number of strips and series terms yields only little improvement.

The results of dimensionless fundamental frequency for different values of aspect ratio b/a and length-to-thickness ratio a/h are shown in Table. 5. And the convergence for long plates ($b/a = 5$, $a/h = 20$) is given in Table 6.

6. *Deflection and free vibration of (0/45/−45/90 degree) laminate*

A square laminate is made up of four equally thick layers oriented at (0/45/−45/90 degrees). Its four edges are hinged and free to move in both tangential and normal inplane directions but fixed at all the corners. The material properties are identical to example 2.

The plate is simulated by four quadratic strips with four series terms.

The resulting central deflection due to uniform load q_0 and the fundamental frequency are listed in Table 7 as functions of the length-to-thickness ratio, a/h.

CONCLUSION

By selecting the proper displacement functions, the present finite strip method includes all the effects of material anisotropy and suits arbitrary inplane boundary conditions at the ends. Thus, this method is applicable not only to cross-ply laminates and symmetrical angle-ply laminates, but also to arbitrary angle-ply laminates, e.g. antisymmetrical angle-ply laminates and (0/45/−45/90 degrees) laminates.

Numerical examples show a close agreement between the present method and the analytical solutions.

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